Explicit Planning for Efficient Exploration in Reinforcement Learning

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ABSTRACT

- Systematic exploration strategies: R-MAX, MBIE, UCRL, their variants, etc. are essentially **heuristic-based**
  - Choose actions greedily w.r.t. some predefined heuristics
  - When heuristics do not match MDP's property well, excessive exploration can happen, reducing learning efficiency
- We propose that **explicit planning for exploration** helps
  - Treat exploration as a two-part procedure:
    1. Evaluate how much data is needed at each state-action pair (= specify an *exploration demand matrix*)
    2. Actually explore and collect data (= fulfil the demand)
  - The second step can be explicitly planned by our Value Iteration for Exploration Cost (VIEC) algorithm
- To show how explicit planning helps, exploration behaviours of ε-greedy, R-MAX, MBIE, and the optimal exploration scheme in **tower MDPs** are analysed and compared
  - Systematic strategies (heuristics): $O(n^2md)$ or $O(n^2m + nmd)$
  - Optimal exploration scheme: $O(nmd)$
  - n = #states, m = #actions, d = demand at each state-action pair
- Exploration behaviour analysis also shows that existing systematic strategies are weak to
  - **Distance traps**: uncertainty being wrongly diminished
  - **Reward traps**: irrelevant rewards that mislead exploration

DEMAND MATRIX

- To guarantee the quality of policy, systematic exploration strategies usually use Hoeffding's or Chernoff's inequalities to evaluate how much data should be collected at each (s, a)
  - Leads to PAC property (sample complexity bounds) or regret bounds
  - R-MAX needs $O(\frac{1}{\varepsilon^2(1-\gamma)^4}(n + \ln\frac{nm}{\delta}))$ data at every (s, a) to be (ε, δ)-PAC [Kakade, 2003; Strehl et al., 2009]
  - MBIE needs $O(\frac{1}{\varepsilon^2(1-\gamma)^4}(n + \ln\frac{nm}{\varepsilon(1-\gamma)\delta}))$ [Strehl and Littman, 2008]
- In practice, obtaining a sufficiently good policy do not need that much data, so people manually tune the exploration parameters to control how much data to be collected
- We call such specifications of data requirement as **(exploration) demands**
  - **Demand matrix** D: entry $D[s, a] = k$ means at least k data should be collected at (s, a) in the following exploration activity
  - For R-MAX with (ε, δ)-PAC property, the elements of the initial demand matrix D_o is uniformly set to $D_o[s, a] = k_0$ for all (s, a) where $k_0 = O(\frac{1}{\varepsilon^2(1-\gamma)^4}(n + \ln\frac{nm}{\delta}))$
  - MBIE & UCRL: although they seem to decide the demand "on the fly" by using confidence intervals (CIs), the frequentist CIs themselves work as *pre-data* analysis, which means that the demand is *already fixed before learning* given the parameter settings (i.e. does not change "on the fly", which is why they can be confidence procedures and thus have performance guarantees)
  - If you already have some prior knowledge to MDP, you can directly specify the demand matrix to reduce unnecessary exploration
  - Simple strategies such as ε-greedy do not have fixed demand matrix and rely on pure luck to find useful information → low efficiency

PLANNING FOR EXPLORATION

- Exploration can be regarded as a two-step procedure:
  - Step1: Specify the exploration demand
  - Step2: Fulfil the demand by collecting data through MDP interaction
  - (optional) jump to Step1 with updated information
- Systematic exploration strategies excel at Step1, but Step2 is treated with less deliberation. That's where *planning* comes in
- Current demand D_t reduces by 1 at (S_t, A_t) after (S_t, A_t) is executed (unless it is already 0) while other elements remain unchanged, i.e.

$$D_{t+1}[s,a] = \begin{cases} \max\{0, D_t[s,a] - 1\}, & (s,a) = (S_t, A_t) \\ D_t[s,a], & \text{otherwise} \end{cases}$$

Such operation is written as $D_{t+1} = H(D_t; S_t, A_t)$
- An **exploration scheme** ψ is a mapping from demand-state pairs to actions, i.e. $\psi(D; s) = a$ indicates action a should be taken when at state s and current demand is D
- **Exploration cost** $C^{\psi}(D; s, a)$ is the expected #steps needed for D to become 0 in an MDP interaction process starting from (s, a) and following exploration scheme ψ
- The planning for exploration problem is an *augmented undiscounted MDP* with:
  - (Augmented) state space $\mathcal{D} \times \mathcal{S}$, action space $\mathcal{A}$
  - Transition $\Pr(D', s', a | D, s) = \begin{cases} P(s'|s,a), & D' = H(D; s, a) \\ 0, & \text{otherwise} \end{cases}$
  - Each step yields cost 1 when $D_t \neq \mathbf{0}$, and cost 0 after $D_t = \mathbf{0}$
- Thus Bellman equation for exploration cost is

$$C^{\psi}(D; s, a) = \begin{cases} 1 + \sum_{s' \in \mathcal{S}} P(s'|s,a) C^{\psi}(D'; s', \psi(D', s')), & D \neq \mathbf{0} \\ 0, & D = \mathbf{0} \end{cases}$$

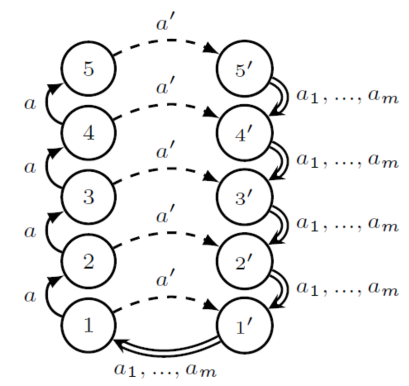
where $D' = H(D; s, a)$.
- We want less exploration cost, so optimal scheme ψ^* has the least $C^{\psi}(D; s, a)$ at every demand-state-action tuple $(D; s, a)$
- **Bellman optimality equation for exploration cost** is

$$C^*(D; s, a) = \begin{cases} 1 + \sum_{s' \in \mathcal{S}} P(s'|s,a) \min_{a' \in \mathcal{A}} C^*(D'; s', a'), & D \neq \mathbf{0} \\ 0, & D = \mathbf{0} \end{cases}$$

where $D' = H(D; s, a)$.
- The optimal exploration scheme ψ^* can be computed by our **Value Iteration for Exploration Cost (VIEC)** algorithm which solves the above equation through a modified Value Iteration process (see paper for detail)
  - Since the demand space $\mathcal{D}$ is exponential in the number of states $|\mathcal{S}|$, the computation of ψ^* is expensive
  - However, most of $\mathcal{D}$ becomes irrelevant after the initial D_0 is given
  - So it should be possible to significantly speed up the computation with techniques such as prioritised sweeping (left to future work)
- VIEC needs to know transition probabilities P to compute ψ^*. When P is unavailable, we can use the estimated transition $\hat{P}$ instead, following an iterative process:
  - Compute exploration scheme $\psi = \mathrm{VIEC}(D, \hat{P})$
  - Collect data by following ψ
  - Update $\hat{P}$ from collected data, jump to the first step with updated $\hat{P}$

TOWER MDP: WHERE HEURISTICS FAIL

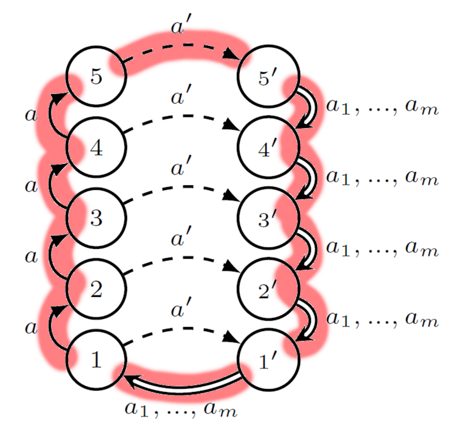
- We use tower MDPs to analyse when and how heuristics fail and planning helps
- A tower MDP of height $h = 5$:



- Upward states $\{s_1, \ldots, s_h\}$, downward states $\{s'_1, \ldots, s'_h\}$
- Taking action a at $s_i \in \{s_1, \ldots, s_{h-1}\}$ goes to s_{i+1} with pr=1
- Taking action a' at $s_i \in \{s_1, \ldots, s_h\}$ goes to s'_i with pr=1
- Each s'_i is an m-armed bandit with unknown reward distributions and leads to s'_{i-1} (or s_1 if $i = 1$)
- Initial demand D_0: uniformly set to a positive interger d for all m-armed bandits, and set to 0 for all (s_i, a) and (s_i, a') due to no uncertainty there

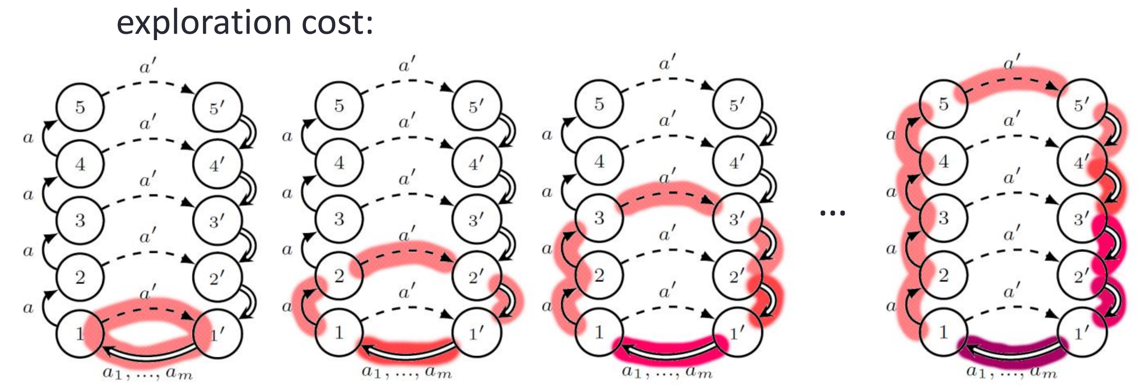
TOWER MDP: OPTIMAL SCHEME

- Optimal scheme: take the marked path $m \times d$ times, each time select a bandit arm with positive demand at the corresponding downward state
- Total exploration cost is $2hmd = \boldsymbol{\Theta(nmd)}$.



TOWER MDP: R-MAX

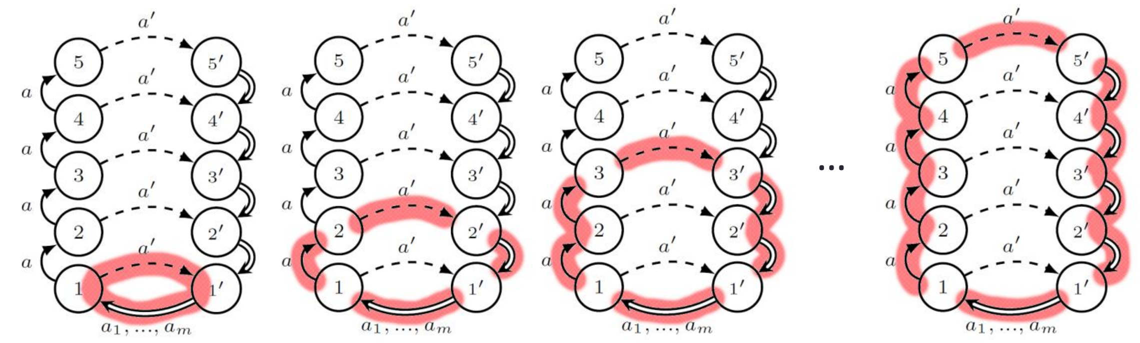
- R-MAX chooses action with highest $\tilde{Q}(s, a)$ which is computed using a modified Bellman equation where "unknown" (s, a) (the ones with positive demand) have $\tilde{Q}(s, a) = \frac{R_{\max}}{1-\gamma}$, which encourages exploration to such (s, a).
- **Distance trap**: due to such design, "uncertainty" is discounted when passed to other states, resulting in R-MAX tending to prioritise the "unknown" state-actions that are near the current state
- In tower MDPs, this results in R-MAX being strongly attracted by the closest "unknown" bandits, greatly increasing the total exploration cost:



- Total exploration cost: $2md + 4md + \cdots + (2h)md = h(h+1)md = \boldsymbol{\Theta(n^2md)}$.

TOWER MDP: MBIE

- MBIE chooses action with highest $\tilde{Q}(s, a)$ which is computed using Bellman equation with modified $\tilde{P}$ and/or $\tilde{R}$ that are the upper bounds of confidence intervals of $\hat{P}$ and $\hat{R}$
- Like R-MAX, MBIE discounts "uncertainty" with distance and thus is weak to the distance traps
- However, the "uncertainty" (represented by $\frac{\beta}{\sqrt{N(s,a)}}$ terms in CIs) diminishes as the number of data $N(s, a)$ increases, which makes MBIE being absorbed less than R-MAX by a nearby "unknown" bandit in tower MDPs:



- In the best case (being trapped by each arm only once), the total exploration cost of MBIE is $(2 + 4 + 6 + \cdots + 2h)m + 2hm(d-1) = \boldsymbol{\Theta(n^2m + nmd)}$
- **Reward trap**: Further, if all bandits give positive rewards, MBIE can be attracted more often than above by the lower-level bandits due to these rewards being considered in $\tilde{Q}$
  - Thus its actual performance will be between $\Theta(n^2m + nmd)$ and $\Theta(n^2md)$
  - Remark: reader may argue that rewards are not always "traps" because they can be designed to guide exploration. However, practice tells us designing a reward function that can properly represent the learning target is already difficult (otherwise we don't need Inverse RL), so designing a reward function that can *both* represent the learning target *and* guide exploration is very difficult. If you try training a Super Mario agent you in this way, you will see Mario ignoring the goal and just trying to get coins infinitely.

CONCLUSION & FUTURE WORK

- Existing systematic strategies are good at specifying the exploration demands, but are weak at fulfilling them
  - Prone to distance & reward traps
- Explicit planning for exploration helps fulfil the exploration demand more efficiently
  - It avoids unnecessary revisits to already explored states that are caused by greedily following predefined heuristics
  - The planning problem can be described as augmented MDPs
  - Optimal exploration scheme exists and can be found by solving Bellman optimality equation for exploration cost
  - Our VIEC algorithm can solve it, though with a high computational cost
- Future work:
  - Fast approximation to VIEC, since the augmented state space is too large and difficult to iterate over
  - Try to classify common MDPs by their dynamic properties, and find out which heuristics are helpful for each class – they can still be helpful when the properties of heuristics and MDP matches